WORK INSTRUCTIONS FOR ENGINEERS

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OP-014. PROCEDURE FOR DETERMINATION OF BENDING MOMENT

14.0 PROCEDURE FOR DETERMINATION OF BENDING MOMENT

14.1. OVERALL PROCEDURES

This section describes the overall procedures for the determination of bending moment based on best-fit curve method on the given deflection profile of a structural element. The procedures are as follows:

1) Determination of curvatures (ψ).
2) Determination of effective moment inertia (I_e).
3) Determination of bending moment (M)..

14.2. DETERMINATION OF CURVATURE (ψ)

1) The measured deflection profile (with all the data points) in the flexural elements, like piles, wall and etc is obtained by curve fitting on the measured displacement of the individual surveyed points.
2) Determine the best-fit curve for a cluster of 7 neighbouring data points with the curvature to be computed at the 4th point of the 7 neighbouring data points by using a six-degree polynomial equation.
3) The first and second derivatives on the above best-fit six-degree polynomial equation at the 4th point yield the value of v' = dy/dx and v" = d^2y/dx^2 respectively.
4) Substitution of these values in the equation below would yield the value of curvature at the point of consideration.

$$\kappa = \frac{1}{\rho} = \frac{v''}{\left[1 + (v')^2\right]^{\frac{3}{2}}}$$

where κ = curvature

ρ = radius of curvature

(Please refer to Appendix I for the derivation and notation for the above equation)

5) The above steps are repeated for the next 7 data points starting from the 2nd to 8th data point and so on until the curvature of all the respective data points along the measured deflection profile are determine.

notes:

(a) A total of 7 data points are considered in a cluster (3 control points on either sides of the fourth point) more or less give a good approximation of curvature at the fourth point. The use of more than 7 data points do not improved further the value obtained.

(b) Currently, most of the available software is capable to fit the data cluster with a polynomial of not more than 10 degrees. Due to such limitation, all the measured data points should not be best-fitted at one go but rather to fit them in several clusters of seven data with six degree polynomial as this is the most practical solution and optimised.

(c) As suggested by Poh(1999), the use of a lower degree curve fitting function, for example a third to fifth degree polynomial, may tend to flatten out some of the local peaks in the curve and hence may underestimate the corresponding curvature. Therefore, a minimum sixth-degree polynomial function is suggested in order to capture the localized larger curve.

14.3. DETERMINATION OF EFFECTIVE MOMENT INERTIA (I_e)

1) Determine gross moment inertia I_g.
2) Determine moment of inertia of a crack transformed section (I_{cr}) at a particular section. Kong and Evans (1987) show that the moment of inertia of a cracked section for a rectangular flexural member can be calculated from the following equation:

$$\frac{I_{cr}}{bd^3} = \frac{1}{3}\left(\frac{x}{d}\right)^3 + \alpha_e \rho\left(1 - \frac{x}{d}\right)^2 + \alpha_e \rho'\left(\frac{x}{d} - \frac{d'}{d}\right)^2$$

in which

$$\frac{I_{cr}}{bd^3} = \frac{1}{3}\left(\frac{x}{d}\right)^3 + \alpha_e \rho\left(1 - \frac{x}{d}\right)^2 + \alpha_e \rho'\left(\frac{x}{d} - \frac{d'}{d}\right)^2$$

where

d = effective depth for the reinforced concrete section

d' = depth from the compression face to the centroid of compression steel.

x = neutral axis

α_e = modular ratio (E_s/E_c)

. ρ = tension steel ratio = A_s/bd

. ρ'= compression steel ratio = A_s'/bd

A_s= tension steel area

A_s'=compression steel area

3) Determine cracking moment (M_{cr})

where

$$M = \frac{f_r I_g}{y_t}$$

in which

f_r = modulus of rupture of concrete

= $0.623\sqrt{f_c'}$ (in MPa)

f_c' = cylinder compressive strength of concrete (in MPa)

= cube strength / 1.25

= (f_{cu}) / 1.25

y_t = the distance from the centroidal axis of cross section, neglecting steel, to the extreme tension fibre

4) Substitute I_{cr}, I_g and M_{cr} in equation 2 below to establish equation of effective moment inertia (I_e).This equation is proposed by Branson (1977):

$$I_e = \left[\frac{M_{cr}}{M}\right]^4 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^4\right] I_{cr} \leq I_g$$

notes: For any reinforced concrete section, the moment of inertia varies with the extent of cracking at the section. When the applied bending moment (M) is sufficiently small where the corresponding outer fibre concrete tensile stress does not exceed the tensile strength or the modulus rupture of concrete, the section will remain uncracked. Under these circumstances, the moment inertia of the uncracked section can be assumed to be the gross moment of inertia (I_g) of a transformed section.

For a cracked section ($M \geq M_{cr}$, where M_{cr} = cracking moment of the concrete section), the moment of inertia of the section is less than the gross moment of inertia and varies with the extent of cracking at the section. However, for the intact concrete block between the cracks, the concrete in tension continues to contribute to the flexural stiffness and therefore reduces the curvature. This phenomenon is called tension stiffening (Ghali 1993).Because of the tension stiffening effect, the actual curvature value at a particular section (partially cracked section) of the intact concrete block between the cracks is smaller than the curvature value for a fully crack section (with the concrete in tension ignored except the transformed area for tension steel still continues to provide tension) but is larger than the curvature value for an uncracked section. Similarly, the moment of inertia for a partially cracked section is less than the gross moment of inertia of the uncracked concrete section but is larger than the moment of inertia for a completely cracked section with the concrete in tension ignored except the tension in tension steel. In this case, an effective moment of inertia (I_e) should be used in the analysis of a cracked section.

14.4. DETERMINATION OF BENDING MOMENT (M)

1) Starting from the gross moment of inertia (Ig) and substitute in the equation below:

$$M = \kappa E_c I.$$

where κ = curvature on the RC section considered

E_c = Young modulus of concrete

I = moment of inertia

2) Substitute M value obtained in step 1 into Eq-2 to determine I_e .

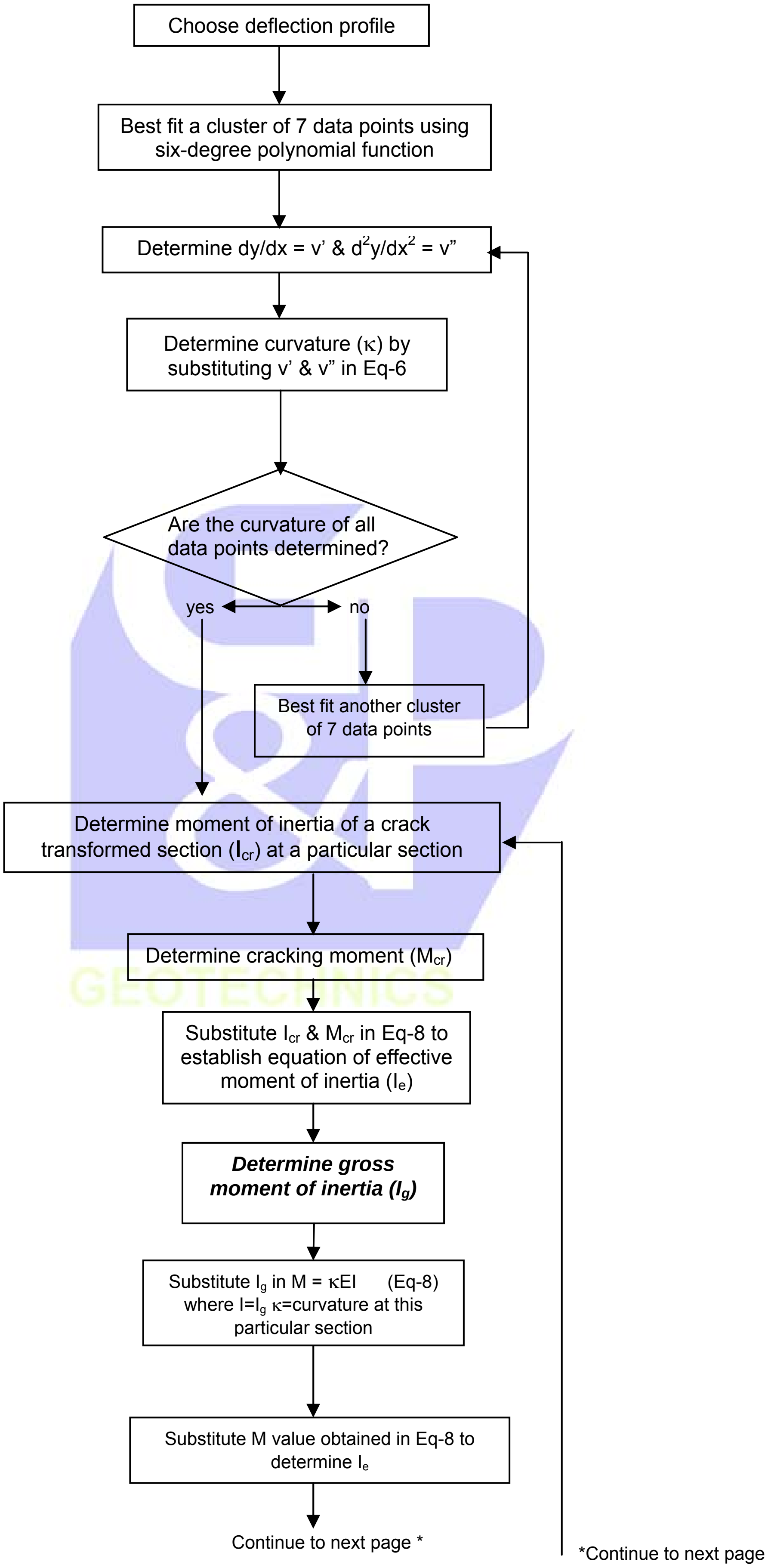
3) Substitute I_e value obtained in step 2 into Eq-3 to determine another value of M

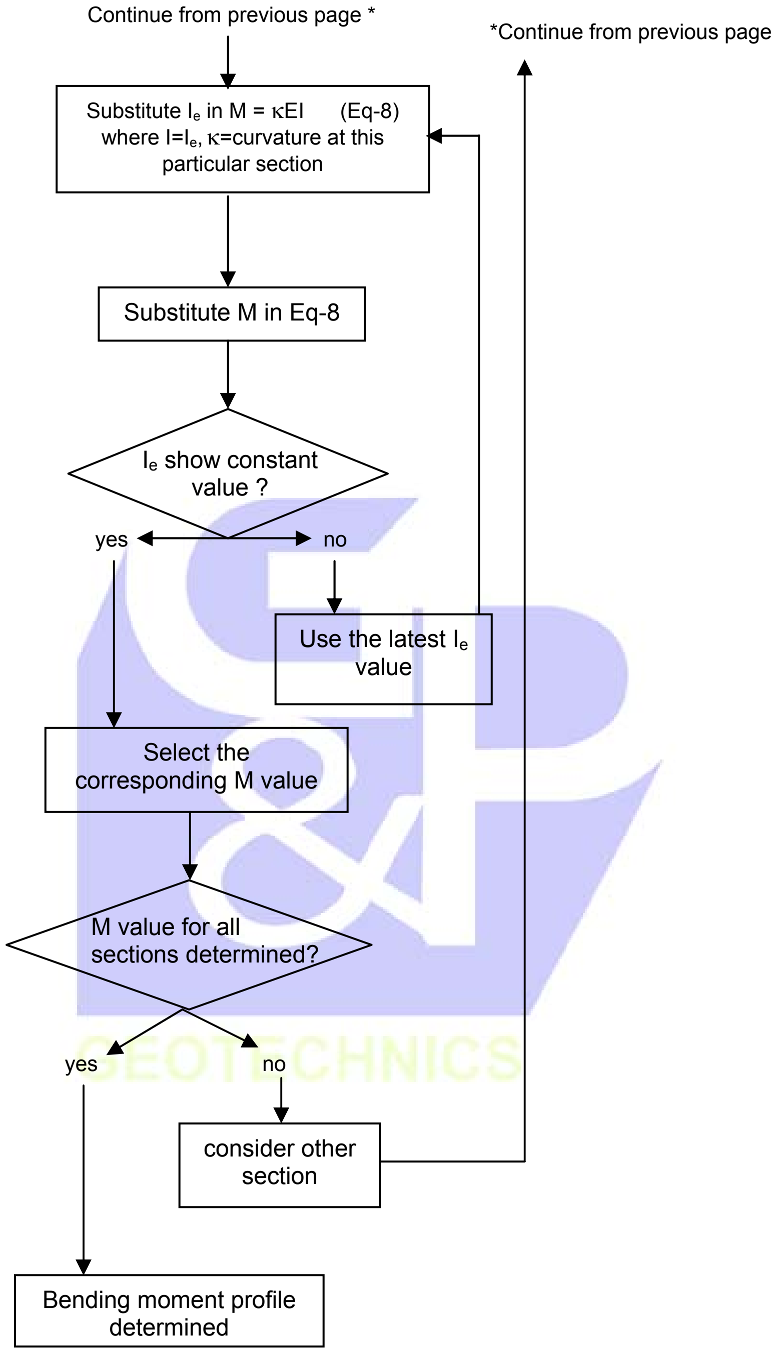
4) Substitute M value obtained in step 3 into Eq-2 to determine another value of I_e.

5) Step 3 and step 4 are repeated until M and I_e show constant value.

6) Steps 1 to 5 will determine the value of moment (when M show constant value) at a particular section of consideration. Repeat the above steps for other section on the flexural member until the bending moment profile are determine.

14.5. FLOW CHART FOR DETERMINATION OF BENDING MOMENT

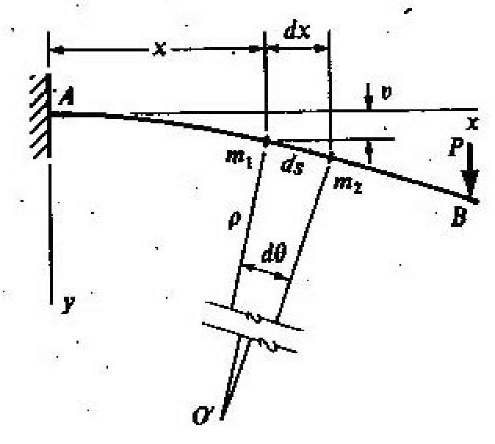



Continue from previous page *
*Continue from previous page
Substitute I_e in $M = \kappa EI$ (Eq-8) where $I=I_e$, κ=curvature at this particular section
Substitute M in Eq-8
I_e show constant value ?
yes
no
Use the latest I_e value
Select the corresponding M value
M value for all sections determined?
yes
no
consider other section
Bending moment profile determined

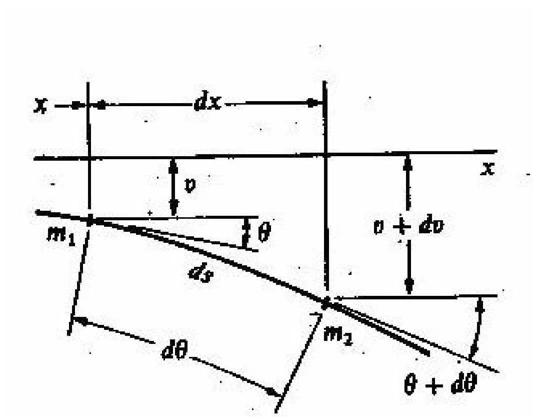
APPENDIX I

14.6. THEORY OF BEAM DEFLECTION

Gere & Timoshenko (1984) have given a very detailed derivation of the beam curvature using the first and second order derivatives of a deflection curve of a beam. Figure 1a shows the details of the beam deflection.



(a)



(b)

Figure 1: Deflection Curve of Beam

If the origin of coordinates is assumed at the fixed end, with the *x*-axis directed to the right and the *y*-axis directed downward. The *xy* plane is a plane of symmetry and all loads act in this plane; thus, the *xy* plane is the plane of bending. The deflection v of the beam at any point m1 at distance *x* from the origin is the translation (or displacement) of that point in the *y* direction, measured from the *x*-axis to the deflection curve.

For the selected axes, a downward deflection is positive and upward deflection is negative, the equation of the deflection curve can be expressed as a function of *x*.

The angle of rotation θ of the axis of the beam at any point m_1 is the angle between the x-axis and the tangent to the deflection curve (Figure 1b). This angle is positive when clockwise, provided the x and y axes have the directions shown.

Now consider a second point m_2, located on the deflection curve at a small distance *ds* further along the curve and at distance $x + dx$ (measured parallel to the x axis) from the origin. The deflection at this point is $v + dv$, where dv is the increment in deflection as we move from m_1 to m_2. Also, the angle of rotation at m_2 is $\theta + d\theta$, where $d\theta$ is the increment in angle of rotation. The intersection of two lines normal to the tangents at points m_1 and m_2 locates the centre of curvature *O'* and the distance from *O'* to the curve is the radius of curvature ρ. From the figure, it can be denoted that $\rho d\theta = ds$; hence, the curvature κ (reciprocal of the radius of curvature) is given by the following equation:

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} \qquad \text{Eq-1}$$

The slope of the deflection curve is the first derivative *dv/dx* and is equal to the tangent of the angle of rotation θ. Due to the reason that *dx* is infinitesimally small; thus

$$\frac{dv}{dx} = \tan\theta \text{ or } \theta = \arctan\frac{dv}{dx} = \arctan v' \qquad \text{Eq-2}$$

From Eq-1 and Eq-2, the expression of curvature can be further derived as follows;

$$\kappa = \frac{1}{\rho} = \frac{d\theta}{ds} = \frac{d(\arctan v')}{dx}\frac{dx}{ds} \qquad \text{Eq-3}$$

From Figure 1b where $ds^2 = dx^2 + dv^2$, the following expression can be obtained;

$$\frac{ds}{dx} = \left[1 + \left(\frac{dv}{dx}\right)^2\right]^{\frac{1}{2}} = \left[1 + (v')^2\right]^{\frac{1}{2}} \qquad \text{Eq-4}$$

By differentiation,

$$\frac{d(\arctan v')}{dx} = \frac{v''}{1 + (v')^2} \qquad \text{Eq-5}$$

Substitution of these expressions (Eq-4 and Eq-5) into the equation for curvature (Eq-3) yields

$$\kappa = \frac{1}{\rho} = \frac{v''}{\left[1 + (v')^2\right]^{\frac{3}{2}}} \qquad \text{Eq-6}$$

It is obvious that the assumption of small slopes is equivalent to disregarding $(v')^2$ in comparison to unity, thus making the denominator in Eq-6 equal to one. where v' and v" denotes dy/dx and d^2y/dx^2 respectively.

Sometimes, the following simplified expression is used for computing curvature of beam undergoing only very small rotation when loaded.

$$v'' = -\frac{R^2}{\left(R^2 - x^2\right)^{\frac{3}{2}}} \qquad \text{Eq-7}$$

This can lead to serious error if the rotation of the deflection beam is large. The following example illustrates the erratic results.

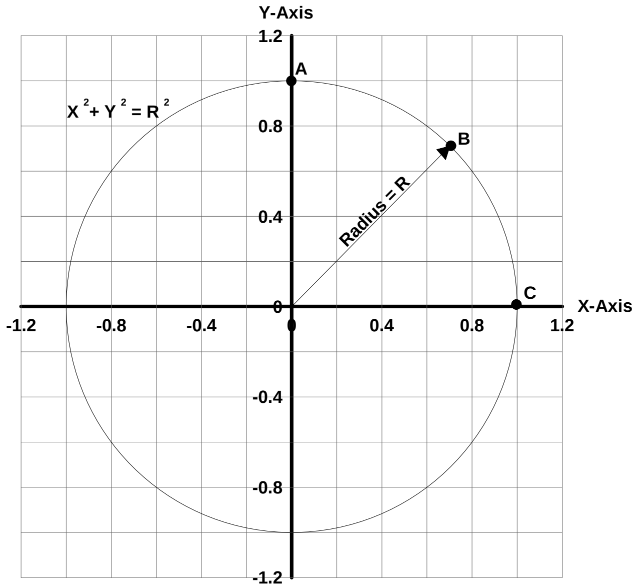


Figure 2: Function of a Circle

Figure 2 shows the function of a circle; the derivations of the curvature are as follows:

- circular equation $y = \sqrt{R^2 - x^2}$
- first derivation $v' = \frac{dy}{dx} = -\frac{x}{\sqrt{R^2 - x^2}}$
- second derivation $v'' = \frac{d^2y}{dx^2} = -\frac{R^2}{\left(R^2 - x^2\right)^{\frac{3}{2}}}$

Point	x	y	v' =dy/dx	v" = d^2y/dx^2	κ $\frac{v''}{\left[1+(v')^2\right]^{\frac{3}{2}}}$
A	0	1.000	0	-1	-1
B	0.707	0.707	-1	-2.828	-1
C	1.000	0	∞	∞	-1/R *

Table 1: Computation of Curvature on Circle

* refer to derivation below

$$v' = \frac{dy}{dx} = -\frac{x}{\sqrt{R^2 - x^2}} \quad \Rightarrow \quad (v')^2 = \frac{x^2}{R^2 - x^2}$$

$$v'' = -\frac{R^2}{\left(R^2 - x^2\right)^{\frac{3}{2}}}$$

$$\kappa = \frac{1}{\rho} = \frac{v''}{\left[1+(v')^2\right]^{\frac{3}{2}}} \qquad = \frac{\dfrac{-R^2}{\left(R^2 - x^2\right)^{3/2}}}{\left[1+\left(\dfrac{x^2}{R^2 - x^2}\right)\right]^{3/2}}$$

$$= \frac{\dfrac{-R^2}{\left(R^2 - x^2\right)^{3/2}}}{\left[\dfrac{R^2 - x^2 + x^2}{\left(R^2 - x^2\right)}\right]^{3/2}} \qquad = \frac{\dfrac{-R^2}{\left(R^2 - x^2\right)^{3/2}}}{\dfrac{R^3}{\left(R^2 - x^2\right)^{3/2}}}$$

$$= -1/R \qquad = -1$$

Table 1 shows the comparison between the simplified expression (Eq-7) and the exact expression (Eq-6) for curvature. It is clear that the exact expression shows the same value at all points. However, the simplified expression gives very erroneous results at point B and C of a circle except at point A where the tangential gradient is parallel to the axis (v' = 0). Therefore, it advisable to use the exact expression for computation of curvature.